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LETTER TO THE EDITOR

Fourier–Gauss transforms of the continuous big q-Hermite polynomials

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Abstract. We examine the Fourier–Gauss transformation properties of a two-parameter family of the continuous big q-Hermite polynomials.

1. Introduction

The goal of this paper is to continue the study of the Fourier–Gauss transformation properties of a family of the five-parameter Askey–Wilson polynomials [1]

$$p_n(x; a, b, c, d|q) := a^{-n}(ab, ac, ad; q)_n {}_4\phi_3 \left[\begin{matrix} q^{-n}, abcdq^{n-1}, a\,\mathrm{e}^{\mathrm{i}\theta}, a\,\mathrm{e}^{-\mathrm{i}\theta} \\ ab, ac, ad, \end{matrix} ; q, q \right] \tag{1.1}$$

in the variable $x = \cos\theta$. The basic hypergeometric series ${}_4\phi_3$ in (1.1) is a particular case of the definition

$${}_r\phi_s \left[\begin{matrix} a_1, \ldots, a_r \\ b_1, \ldots, b_n \end{matrix} ; q, z \right] := \sum_{k=0}^{\infty} \frac{(a_1, a_2, \ldots, a_r; q)_k}{(b_1, b_2, \ldots, b_s, q; q)_k} [(-1)^k q^{k(k-1)/2}]^{1+s-r} z^k \tag{1.2}$$

with the standard notation of q-analysis [2]

$$(a; q)_k = \prod_{j=0}^{k-1}(1 - aq^j) \qquad (a_1, \ldots, a_r; q)_k = \prod_{j=1}^{k}(a_j; q)_k. \tag{1.3}$$

Observe that if one of the parameters a_j, $1 \leqslant j \leqslant r$, in (1.2) is equal to q^{-n}, $n = 1, 2, 3, \ldots$, then $(q^{-n}; q)_k = 0$ for $k \geqslant n + 1$ by definition (1.3) and the corresponding series ${}_r\phi_s$ is therefore represented by a finite sum in k from zero to n.

The Askey–Wilson polynomials (1.1) are symmetric with respect to the parameters a, b, c, d and

$$p_n(-x; a, b, c, d|q) = (-1)^n p_n(x; -a, -b, -c, -d|q). \tag{1.4}$$

The Askey–Wilson polynomials (1.1) with vanishing parameters a, b, c and d correspond to the continuous q-Hermite polynomials

$$H_n(x|q) := p_n(x; 0, 0, 0, 0|q) \tag{1.5}$$

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of Rogers [3, 4]. At the next level of the Askey–Wilson family (1.1) one of the parameters a, b, c, and d is non-vanishing, while three others are equal to zero. This special case defines the continuous big q-Hermite polynomials [5–7]

$$H_n(x; a|q) := p_n(x; a, 0, 0, 0|q). \tag{1.6}$$

The Fourier–Gauss transformation properties of the continuous q-Hermite polynomials (1.5) have been studied in [8]. Section 2 collects mostly known results about these polynomials, which are needed in section 3 to derive Fourier–Gauss transforms of the continuous big q-Hermite polynomials (1.6).

2. The *q*-Hermite ladder

The explicit form of the continuous q-Hermite polynomials $H_n(x|q)$ is given by their Fourier expansion

$$H_n(\sin\kappa s|q) = \mathrm{i}^n \sum_{k=0}^{n} (-1)^k \begin{bmatrix} n \\ k \end{bmatrix}_q \mathrm{e}^{\mathrm{i}(2k-n)\kappa s} = \mathrm{i}^n\, \mathrm{e}^{-\mathrm{i}n\kappa s}\, {}_2\phi_0(q^{-n}, 0; q, -q^n\, \mathrm{e}^{2\mathrm{i}\kappa s}) \tag{2.1}$$

where $\left[\begin{smallmatrix} n \\ k \end{smallmatrix}\right]_q$ is the q-binomial coefficient,

$$\begin{bmatrix} n \\ k \end{bmatrix}_q := \frac{(q;q)_n}{(q;q)_k (q;q)_{n-k}} = \begin{bmatrix} n \\ n-k \end{bmatrix}_q . \tag{2.2}$$

This follows from the definitions (1.1) and (1.5) after employing the identity [2]

$$(q^{-n}; q)_k = (-1)^k q^{k(k-1)/2-nk} \frac{(q;q)_n}{(q;q)_{n-k}} \tag{2.3}$$

and the relation (see [5, p 18, formula (0.6.28)])

$${}_3\phi_2(q^{-n}, a, b; 0, 0; q, q) = a^n\, {}_2\phi_0(q^{-n}, a; q, bq^n/a). \tag{2.4}$$

Observe that for our purposes we find it more appropriate to use the parametrization $x = x_q(s) := \sin\kappa s, q = \exp(-2\kappa^2)$, which is equivalent to the change of variables $\theta = \pi/2 - \kappa s$ in (1.1).

In the limit case when the parameter q tends to 1 (and, consequently, $\kappa \to 0$) we have

$$\lim_{q\to 1} \kappa^{-n} H_n(\sin\kappa s|q) = H_n(s) \tag{2.5}$$

where $H_n(s)$ are the ordinary Hermite polynomials.

One can also consider the continuous q^{-1}-Hermite polynomials [4]

$$h_n(x|q) := \mathrm{i}^{-n} H_n(\mathrm{i}x|q^{-1}) \tag{2.6}$$

by transforming $q \to q^{-1}$ in (1.5). Because of the inversion formula

$$\begin{bmatrix} n \\ k \end{bmatrix}_{q^{-1}} = q^{k(k-n)} \begin{bmatrix} n \\ k \end{bmatrix}_q \tag{2.7}$$

from (2.1) it follows that their explicit form is

$$h_n(\sinh\kappa s|q) = \sum_{k=0}^{n} (-1)^k q^{k(k-n)} \begin{bmatrix} n \\ k \end{bmatrix}_q \mathrm{e}^{(n-2k)\kappa s} = \mathrm{e}^{n\kappa s}\, {}_1\phi_1(q^{-n}; 0; q, -\mathrm{e}^{-2\kappa s}). \tag{2.8}$$

The continuous q-Hermite (2.1) and q^{-1}-Hermite (2.7) polynomials are related to each other through the Fourier–Gauss integral transform [8]

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} H_n(\sin\kappa s|q)\, \mathrm{d}s = \mathrm{i}^n q^{n^2/4} h_n(\sinh\kappa r|q)\, \mathrm{e}^{-r^2/2}. \tag{2.9}$$

As is evident from (2.5), in the limit when $q \to 1$ this integral transform reduces to that for the Hermite polynomials $H_n(s)$:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} H_n(s)\, \mathrm{d}s = \mathrm{i}^n H_n(r)\, \mathrm{e}^{-r^2/2}. \tag{2.10}$$

It is essential to note that one may consider another parametrization $x = y_q(s) := \cos \kappa s$ for the argument of the q-Hermite polynomials in (2.1), which is linearly independent from $x_q(s) = \sin \kappa s$. The corresponding Fourier–Gauss transform has the form

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} H_n(\cos \kappa s|q)\, \mathrm{d}s = q^{n^2/4} H_n(\cos \kappa r|q^{-1})\, \mathrm{e}^{-r^2/2}. \tag{2.11}$$

Since

$$\lim_{q \to 1} H_n(\cos \kappa s|q) = 2^n \tag{2.12}$$

in the limit as $q \to 1$ the integral transform (2.11) only reproduces the particular case of (2.10) with $n = 0$, that is, the Fourier transform for the Gauss exponential function $\exp(-s^2/2)$. This circumstance explains why it is more interesting to consider the parametrization $x_q(s) = \sin \kappa s$.

3. The big q-Hermite ladder

The next level of the Askey–Wilson hierarchy (1.1) corresponds to a two-parameter family of the continuous big q-Hermite polynomials

$$\begin{aligned} H_n(\sin \kappa s; a|q) &= a^{-n}\, {}_3\phi_2(q^{-n}, \mathrm{i}a\, \mathrm{e}^{-\mathrm{i}\kappa s}, -\mathrm{i}a\mathrm{e}^{\mathrm{i}\kappa s}; 0, 0; q, q) \\ &= \mathrm{i}^n\, \mathrm{e}^{-\mathrm{i}n\kappa s}\, {}_2\phi_0(q^{-n}, \mathrm{i}a\, \mathrm{e}^{-\mathrm{i}\kappa s}; q, -q^n\, \mathrm{e}^{2i\kappa s}). \end{aligned} \tag{3.1}$$

The second line in (3.1) follows from the relation (2.4) between the terminating basic hypergeometric series ${}_3\phi_2$ and ${}_2\phi_0$.

Two neighbouring levels (2.1) and (3.1) are explicitly related by

$$H_n(x; a|q) = \sum_{k=0}^{n} q^{k(k-1)/2} \begin{bmatrix} n \\ k \end{bmatrix}_q (-a)^k H_{n-k}(x|q). \tag{3.2}$$

The coefficients of $H_{n-k}(x|q)$ in (3.2) are a special case of the general formula for the connection coefficients of the Askey–Wilson polynomials (1.1), derived in [1] (see also [7]). The inverse expansion with respect to (3.2) is

$$H_n(x|q) = \sum_{k=0}^{n} a^{n-k} \begin{bmatrix} n \\ k \end{bmatrix}_q H_k(x; a|q). \tag{3.2$'$}$$

To verify (3.2′), multiply both sides of (3.2) by the factor $\left[\begin{smallmatrix} m \\ n \end{smallmatrix}\right]_q a^{-n}$ and sum over n from zero to m. This gives (3.2′) upon employing the orthogonality relation [9]

$$\sum_{k=0}^{m} (-1)^k q^{k(k-1)/2} \begin{bmatrix} m \\ k \end{bmatrix}_q \begin{bmatrix} m-k \\ n \end{bmatrix}_q = \delta_{mn} \tag{3.3}$$

for the q-binomial coefficients (2.2).

From the limit relation [5]

$$\lim_{q \to 1} \kappa^{-n} H_n(\kappa s; 2\kappa a|q) = H_n(s - a) \tag{3.4}$$

it follows that when $q \to 1$ the expansions (3.2) and (3.2′) reduce to the identities [7]

$$H_n(s-a) = \sum_{k=0}^{n} (-2a)^{n-k} \binom{n}{k} H_k(s) \tag{3.5}$$

$$H_n(s) = \sum_{k=0}^{n} (2a)^{n-k} \binom{n}{k} H_k(s-a) \tag{3.5′}$$

for the classical Hermite polynomials, respectively. Here $\binom{n}{k}$ is the ordinary binomial coefficient.

In analogy with (2.6), one can introduce the continuous big q^{-1}-Hermite polynomials

$$h_n(x;a|q) := \mathrm{i}^{-n} H_n(\mathrm{i}x;a|q^{-1}) = q^{-n(n-1)/2} \sum_{k=0}^{n} q^{k(k-1)/2} \begin{bmatrix} n \\ k \end{bmatrix}_q (\mathrm{i}a)^{n-k} h_k(x|q). \tag{3.6}$$

Their explicit form is

$$\begin{aligned} h_n(\sinh\kappa s;a|q) &= (\mathrm{i}a)^{-n} {}_3\phi_0(q^{-n}, \mathrm{i}\,\mathrm{e}^{\kappa s}/a, -\mathrm{i}\,\mathrm{e}^{-\kappa s}/a; q, a^2q^n) \\ &= \mathrm{e}^{n\kappa s} {}_2\phi_1(q^{-n}, -\mathrm{i}\,\mathrm{e}^{-\kappa s}/a; 0; q, -\mathrm{i}aq\,\mathrm{e}^{-\kappa s}). \end{aligned} \tag{3.7}$$

The second line in (3.7) follows from the equality

$${}_2\phi_1(q^{-n}, b; 0; q, z) = b^n {}_3\phi_0(q^{-n}, b, q/z; q, q^{n-1}z/b) \tag{3.8}$$

which is a particular case of the transformation for a terminating series ${}_2\phi_1$ with a vanishing parameter c (see [2], formula (III.8)).

The inverse relation with respect to (3.6) is

$$h_m(x|q) = \sum_{n=0}^{m} q^{n(n-m)} \begin{bmatrix} m \\ n \end{bmatrix}_q (-\mathrm{i}a)^{m-n} h_n(x;a|q). \tag{3.6′}$$

In the limit case when the parameter q tends to 1, we have

$$\lim_{q\to 1} \kappa^{-n} h_n(\kappa s; 2\kappa a|q) = H_n(s+\mathrm{i}a). \tag{3.9}$$

The first type of the Fourier–Gauss transform

$$\begin{aligned} &\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} H_n(\sin\kappa s; a|q)\,\mathrm{d}s \\ &\quad = \mathrm{i}^n q^{n^2/4}\,\mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{3k^2/4-(n+1)k/2} \begin{bmatrix} n \\ k \end{bmatrix}_q (\mathrm{i}a)^k h_{n-k}(\sinh\kappa r|q) \end{aligned} \tag{3.10}$$

for the continuous big q-Hermite polynomials (3.1) is an immediate consequence of (3.2) and (2.9). Substituting (3.6′) into the right-hand side of (3.10), one can also represent (3.10) alternatively as

$$\begin{aligned} &\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} H_n(\sin\kappa s; a|q)\,\mathrm{d}s \\ &\quad = \mathrm{i}^n q^{n^2/4}\,\mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{k(k-n)} \begin{bmatrix} n \\ k \end{bmatrix}_q c_{k,n}(q)(-\mathrm{i}a)^k h_{n-k}(\sinh\kappa r; a|q) \end{aligned} \tag{3.10′}$$

where the a-independent constant $c_{k,n}(q)$ is equal to

$$c_{k,n}(q) = \sum_{j=0}^{k} (-1)^j q^{3j^2/4 - kj + (n-1)j/2} \begin{bmatrix} k \\ j \end{bmatrix}_q = \sum_{j=0}^{k} \frac{(q^{-k};q)_j}{(q;q)_j} q^{(j/2+n)j/2}. \tag{3.11}$$

The Fourier–Gauss integrals (3.10) and (3.10′) thus transform the continuous big q-Hermite polynomials (3.1) into linear combinations of either the q^{-1}-Hermite polynomials (i.e. from the lower level of the Askey–Wilson hierarchy) or the big q^{-1}-Hermite polynomials (i.e. of the same level), respectively.

In a similar manner, by using the inverse to the (2.9) Fourier–Gauss transform and the relations (3.2′) and (3.6), one obtains

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}sr-s^2/2} h_n(\sinh\kappa s; a|q)\,\mathrm{d}s$$

$$= \mathrm{i}^{-n} q^{-n^2/4}\,\mathrm{e}^{-r^2/2} \sum_{k=0}^{n} q^{k^2/4+(1-n)k/2} \begin{bmatrix} n \\ k \end{bmatrix}_q (-a)^k H_{n-k}(\sin\kappa r|q) \tag{3.12}$$

$$= \mathrm{i}^{-n} q^{-n^2/4}\,\mathrm{e}^{-r^2/2} \sum_{k=0}^{n} c_{k,n}(q^{-1}) \begin{bmatrix} n \\ k \end{bmatrix}_q a^k H_{n-k}(\sin\kappa r; a|q). \tag{3.12′}$$

Observe that the integral transforms (3.10) and (3.12), as well as (3.10′) and (3.12′), are related to each other by a replacement of the base $q \to q^{-1}$ (i.e. $\kappa \to \mathrm{i}\kappa$). In the limit case when the parameter q tends to 1, these integral transforms coincide with

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr-s^2/2} H_n(s-a)\,\mathrm{d}s = \mathrm{i}^n H_n(r+\mathrm{i}a)\,\mathrm{e}^{-r^2/2} \tag{3.13}$$

and its inverse, respectively.

A derivation of another type of Fourier–Gauss integral transform for the continuous big q-Hermite polynomials (3.1) is based upon the relation [10]

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr-s^2/2} e_q(t\,\mathrm{e}^{\mathrm{i}\kappa s})\,\mathrm{d}s = \varepsilon_q(t\,\mathrm{e}^{-\kappa r})\,\mathrm{e}^{-r^2/2} \tag{3.14}$$

between the q-exponential functions

$$e_q(z) := \sum_{n=0}^{\infty} \frac{z^n}{(q;q)_n} \qquad \varepsilon_q(z) := \sum_{n=0}^{\infty} \frac{q^{n^2/4}}{(q;q)_n} z^n. \tag{3.15}$$

Indeed, let us consider an integral

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr-s^2/2} H_n(\sin\kappa s; a|q) e_q(\mathrm{i}a\,\mathrm{e}^{-\mathrm{i}\kappa s})\,\mathrm{d}s. \tag{3.16}$$

Substituting the second line from (3.1) into (3.16) gives the expression

$$\frac{\mathrm{i}^n}{\sqrt{2\pi}} \sum_{k=0}^{n} \frac{(q^{-n};q)_k}{(q;q)_k} q^{nk-k(k-1)/2} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}[r+(2k-n)\kappa]s-s^2/2} (\mathrm{i}a\,\mathrm{e}^{-\mathrm{i}\kappa s};q)_k e_q(\mathrm{i}a\,\mathrm{e}^{-\mathrm{i}\kappa s})\,\mathrm{d}s. \tag{3.17}$$

Since

$$(z;q)_k e_q(z) = e_q(q^k z) \tag{3.18}$$

one can rewrite (3.17) as

$$\frac{\mathrm{i}^n}{\sqrt{2\pi}} \sum_{k=0}^{n} \frac{(q^{-n};q)_k}{(q;q)_k} q^{nk-k(k-1)/2} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}[r+(2k-n)\kappa]s-s^2/2} e_q(\mathrm{i}aq^k\,\mathrm{e}^{-\mathrm{i}\kappa s})\,\mathrm{d}s. \tag{3.19}$$

The integral in (3.19) is now evaluated by the aid of (3.14). We thus arrive at a Fourier–Gauss transform of the form

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr-s^2/2} H_n(\sin\kappa s; a|q) e_q(\mathrm{i}a\,\mathrm{e}^{-\mathrm{i}\kappa s})\,\mathrm{d}s = \mathrm{i}^n q^{n^2/4} h_n(\sinh\kappa r|q) \varepsilon_q(\mathrm{i}aq^{n/2}\,\mathrm{e}^{\kappa r})\,\mathrm{e}^{-r^2/2}. \tag{3.20}$$

The inverse integral transform is

$$\frac{\mathrm{i}^n q^{n^2/4}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}sr - s^2/2} h_n(\sinh \kappa r|q) \varepsilon_q(\mathrm{i}aq^{n/2}\,\mathrm{e}^{\kappa r})\,\mathrm{d}r = H_n(\sin \kappa s; a|q) e_q(\mathrm{i}a\,\mathrm{e}^{-\mathrm{i}\kappa s})\,\mathrm{e}^{-s^2/2}. \tag{3.20$'$}$$

It is obvious that for $a = 0$ the integral transforms (3.20) and (3.20′) coincide with (2.9) and its inverse, respectively.

Similarly, using the Fourier–Gauss integral transform [10]

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} E_q(t\,\mathrm{e}^{\kappa s})\,\mathrm{d}s = \varepsilon_q(-q^{-1/2} t\,\mathrm{e}^{\mathrm{i}\kappa r})\,\mathrm{e}^{-r^2/2} \tag{3.21}$$

for the reciprocal to the $e_q(z)$ q-exponential function

$$E_q(z) := \sum_{n=0}^{\infty} \frac{q^{n(n-1)/2}}{(q;q)_n} (-z)^n \tag{3.22}$$

and the relation

$$(q/z;q)_n E_q(z) = q^{n(n+1)/2} (-z)^{-n} E_q(q^{-n} z) \tag{3.23}$$

one obtains

$$\frac{\mathrm{i}^{-n} q^{n^2/4}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}sr - s^2/2} h_n(\sinh \kappa s; a|q) E_q(\mathrm{i}aq\,\mathrm{e}^{\kappa s})\,\mathrm{d}s = H_n(\sin \kappa r|q) \varepsilon_q(-\mathrm{i}aq^{(1-n)/2}\,\mathrm{e}^{\mathrm{i}\kappa r})\,\mathrm{e}^{-r^2/2}. \tag{3.24}$$

Its inverse integral transform is

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}sr - s^2/2} H_n(\sin \kappa r|q) \varepsilon_q(-\mathrm{i}aq^{(1-n)/2}\,\mathrm{e}^{\mathrm{i}\kappa r})\,\mathrm{d}r = \mathrm{i}^{-n} q^{n^2/4} h_n(\sinh \kappa s; a|q) E_q(\mathrm{i}aq\,\mathrm{e}^{\kappa s})\,\mathrm{e}^{-s^2/2}. \tag{3.24$'$}$$

Observe that since

$$e_{1/q}(z) = E_q(-qz) \qquad \varepsilon_{1/q}(z) = \varepsilon_q(-q^{1/2} z) \tag{3.25}$$

the Fourier–Gauss integral transforms (3.24) and (3.24′) are related to (3.20) and (3.20′), respectively, by a replacement of the base $q \to q^{-1}$ (i.e. $\kappa \to \mathrm{i}\kappa$). A common characteristic feature to note about all of these four integral transforms is that they connect polynomial families from neighbouring levels (2.1) and (3.1) of the Askey–Wilson hierarchy (1.1) (cf formula (2.9)).

4. Concluding remarks

Once the Fourier–Gauss transform (2.9) between the continuous q-Hermite polynomials $H_n(x|q)$ with different values of the parameter q [8] and a Ramanujan-type continuous measure of orthogonality [11] for the Askey–Wilson polynomials $p_n(x; a, b, c, d|q)$ were established, it became clear that the same type of integral transforms might exist for the higher levels of the Askey–Wilson hierarchy. In the present paper we have been able to verify this conjecture [11] for the continuous big q-Hermite polynomials $H_n(x; a|q)$, which are one step higher than $H_n(x|q)$. The possibility of finding Fourier–Gauss transforms for the next levels of the Askey–Wilson family is of great interest for a deeper understanding of the classical q-orthogonal polynomials and certain questions about their classification. Work in this direction is in progress.

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